Exercise 323

Solve the following trigonometric equations on the interval $\theta = [-2\pi, 2\pi]$ exactly.

 $\sec^2 x - 2\sec x + 1 = 0$

Solution

Notice that the left side can be factored.

$$(\sec x - 1)^2 = 0$$

The quantity in parentheses must be zero.

 $\sec x - 1 = 0$

Isolate the term with x.

$$\sec x = 1$$

Secant is the reciprocal of cosine.

$$\frac{1}{\cos x} = 1$$

Multiply both sides by $\cos x$.

 $\cos x = 1$

The general solution is $x = 2\pi n$, where n is any integer. Only three values of x lie between -2π to 2π .

$$x = \{-2\pi, 0, 2\pi\}$$